# ME 314 - Engineering Design : Mechanical Components
# Lecture 19

Note Title

## 10.7 Shaft Failure in Combined Loading

**Results of fatigue tests on steel specimens subjected to combined torsion and bending are shown in Fig. 10-3. As shown, the data generally follow the elliptical relationships defined in the figure. Cast brittle materials (not shown), on the other hand, were found to fail based on the maximum principal stress theory.**

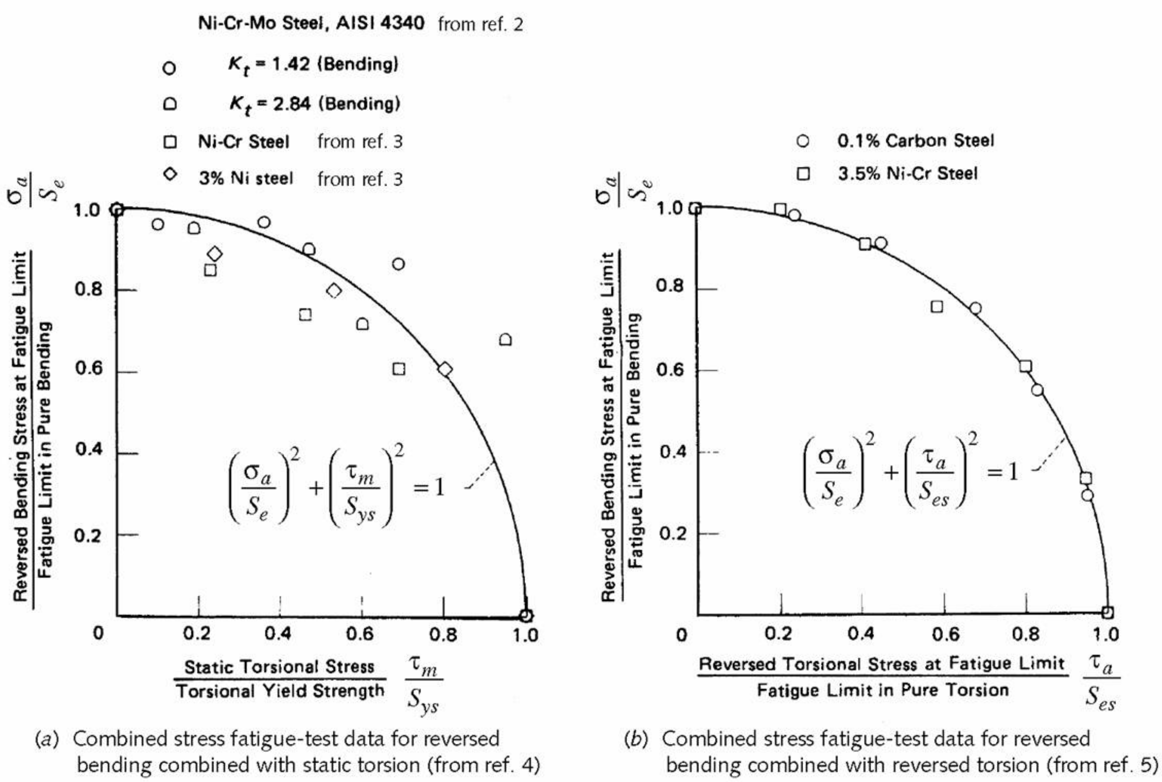



Figure 10-3

Results of Fatigue Tests of Steel Specimens Subjected to Combined Bending and Torsion.
(From *Design of Transmission Shafting,* American Society of Mechanical Engineers, New York, ANSI/ASME Standard B106.1M-1985, with permission).

## 10.8 Shaft Design

**Both stresses and deflections need to be considered. Since the stresses can be calculated locally for various points along the shaft based on known loads and assumed cross sections while the deflection calculations require that the entire shaft geometry be defined, the shaft is first designed using stress considerations and then the deflections are calculated once the geometry is completely defined.**

### Design for Fully Reversed Loading and Steady Torsion

**In this case, the alternating component of torsional stress is absent. Experimental data for this case are shown in Fig. 10-3a and an ASME method (B106.1M-1985) for design of shafts under this type of loading has been published. The failure envelope as shown in Fig. 10-3a can be expressed by the equation:**

$$\left(\frac{\sigma_a}{S_e}\right)^2+\left(\frac{\tau_m}{S_{ys}}\right)^2=1 \qquad (10.5a)$$

**Introducing a safety factor, $N_f$:**

$$\left(N_f\frac{\sigma_a}{S_e}\right)^2+\left(N_f\frac{\tau_m}{S_{ys}}\right)^2=1 \qquad (10.5b)$$

**Using the von Mises relationship for $S_{ys}$ (Eq. 5.9, p. 251):**

$$S_{ys}=\frac{S_y}{\sqrt{3}} \qquad (10.5c)$$

**we have, after substituting for $\sigma_a$ and $\tau_m$ from 10.2c and 10.3c:**

$$\left[\left(k_f\frac{32M_a}{\pi d^3}\right)\left(\frac{N_f}{S_e}\right)\right]^2+\left[\left(k_{fsm}\frac{16T_m}{\pi d^3}\right)\left(\frac{N_f\sqrt{3}}{S_y}\right)\right]^2=1 \qquad (10.5e)$$

**which can be solved for the shaft diameter, d, as**

$$d=\left\{\frac{32N_f}{\pi}\left[\left(k_f\frac{M_a}{S_f}\right)^2+\frac{3}{4}\left(k_{fsm}\frac{T_m}{S_y}\right)^2\right]^{\frac{1}{2}}\right\}^{\frac{1}{3}} \qquad (10.6a)$$

**Fig. 10-4 shows the elliptical failure line used in this approach.**

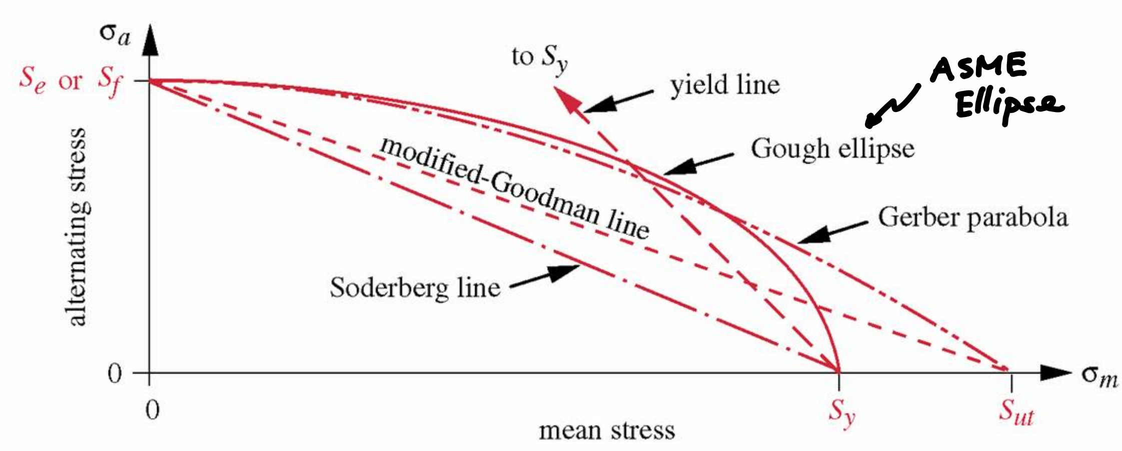



Figure ~~9~~ 10-4

Elliptical Failure Line Using Yield Strength Shown with Other Failure Lines for Fluctuating Stresses.

**Since the ASME ellipse is less conservative than the modified-Goodman, instead of Eq. (10.6a), it is better to use the more general Eq. 10.8 (see below).**

## Design for Fluctuating Bending and Fluctuating Torsion

**The approach is as described in Section 6.12 (P. 376). The stress state is two-dimensional so we have:**

$$\sigma_a' = \sqrt{\sigma_a^2 + 3\tau_a^2} \quad , \quad \sigma_m' = \sqrt{(\sigma_m + \sigma_{m\,axial})^2 + 3\tau_m^2} \qquad (10.7a)$$

**These von Mises stresses can now be entered into a modified-Goodman diagram (MGD)) to find a F.S., or we can use Eq. 6.18 (p.367-368) directly. If the mean and alternating loads maintain a constant ratio, Eq. (6.18 e) for Case 3 holds:**

$$N_f = \frac{S_f S_{ut}}{\sigma_a' S_{ut} + \sigma_m' S_f} \quad \text{or} \quad \frac{1}{N_f} = \frac{\sigma_a'}{S_f} + \frac{\sigma_m'}{S_{ut}} \qquad (10.7b)$$

**Now employing 10.7a, 10.2c, and 10.3c, $\sigma'_a$ and $\sigma'_m$ are given by**

$$\sigma_a' = \left[\left(k_f \frac{32 M_a}{\pi d^3}\right)^2 + 3\left(k_{fs} \frac{16 T_a}{\pi d^3}\right)^2\right]^{1/2}$$

$$\sigma_m' = \left[\left(k_f \frac{32 M_m}{\pi d^3}\right)^2 + 3\left(k_{fsm} \frac{16 T_m}{\pi d^3}\right)^2\right]^{1/2}$$

**where $\sigma_{m\,axial} = 0$ has been assumed. Substituting from these into Eq. 10.7b, we obtain**

$$\frac{1}{N_f} = \frac{32}{\pi d^3}\left\{\frac{1}{S_f}\left[(k_f M_a)^2 + \frac{3}{4}(k_{fsa} T_a)^2\right]^{1/2} + \frac{1}{S_{ut}}\left[(k_{fm} M_m)^2 + \frac{3}{4}(k_{fsm} T_m)^2\right]^{1/2}\right\}$$

or

$$d = \left\{\frac{32 N_f}{\pi}\left[\frac{1}{S_f}\left[(k_f M_a)^2 + \frac{3}{4}(k_{fsa} T_a)^2\right]^{1/2} + \frac{1}{S_{ut}}\left[(k_{fm} M_m)^2 + \frac{3}{4}(k_{fsm} T_m)^2\right]^{1/2}\right]\right\}^{1/3} \qquad (10.8)$$

**This is the equation to use for design of shafts when axial stress is zero and Case 3 for loading holds.**

**Reading Assignment: Sections 10.9 - 10.11.**

# Top Ten Things to Consider in Designing Shafts

**1. To minimize both deflections & stresses, the shaft length should be kept as short as possible and overhangs must be minimized.**

**2. Unless a cantilever shaft is dictated by design constraints, it should be avoided because it has a larger deflection than the simply supported shaft (for the same load, length, and cross section).**

**3. A hollow shaft has a better stiffness/mass ratio and higher natural frequencies than a comparably stiff or strong solid shaft, but will be more expensive and larger in diameter.**

**4. Try to locate stress-raisers away from regions of large bending moment if possible and minimize their effects with generous radii and reliefs.**

**5. If minimizing deflection is the primary concern, then low-carbon steel may be the preferred material since its stiffness is as high as that of more expensive steels and a shaft designed for low deflection will tend to have low stresses.**

**6. Deflections at gears carried on the shaft should not exceed about 0.005 in and the relative slope between the gear axes should be less than about 0.030°.**

**7. If sleeve bearings are used, the shaft deflection across the bearing length should be less than the oil-film thickness in the bearing.**

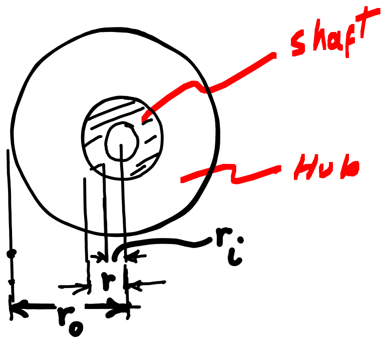
**8. If non-self-aligning rolling element bearings are used, the shaft's slope at the bearings should be kept less than about 0.04°.**

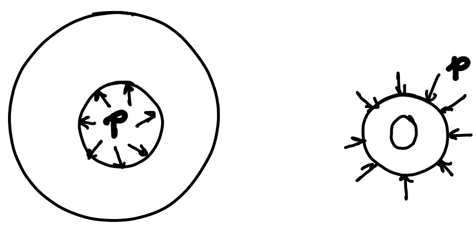
**9. If axial thrust loads are present, they should be carried by *one and only one* bearing in each direction. Otherwise, thermal expansion of the shaft can overload the bearings.**

**10. The first natural frequency of the shaft should be at least three times the highest forcing frequency expected in service, and preferably much more.**

### 10.12 Interference Fits

**Interference fits which are also known as press or shrink fits are a common way of mounting a hub on a shaft.**



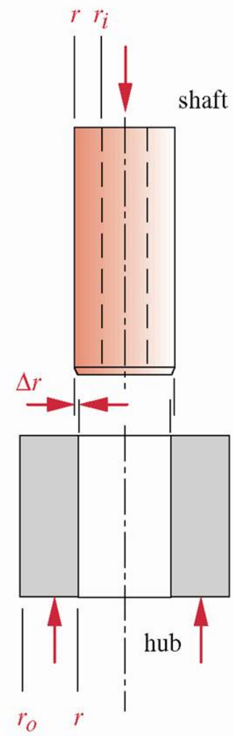
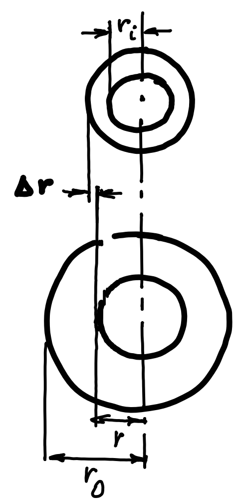




Figure ~~9-18~~ 10-18
An Interference Fit.

**The pressure p created by the press fit can be found from the deformation of the materials caused by the interference:**

$$p = \frac{0.5\,\delta}{\frac{r}{E_o}\left(\frac{r_o^2 + r^2}{r_o^2 - r^2} + \nu_o\right) + \frac{r}{E_i}\left(\frac{r^2 + r_i^2}{r^2 - r_i^2} - \nu_i\right)} \qquad (10.14a)$$




**where $\delta = 2\Delta$ is the total diameteral interference, r is the nominal radius of the interface between the parts, $r_i$ is the inside radius (if any) of a hollow shaft, and $r_0$ is the outside radius of the hub E and $\nu$ are the Young's modulus and Poisson's ratio of the materials.**

**The torque transmitted by the interference fit is obtained in terms of the pressure at the interface, p, which creates a friction force at the shaft radius:**

$$T = 2\pi r^2 \mu p l \qquad (10.14b)$$

**where l is the length of the hub engagement, r is the shaft radius, and $\mu$ is the coefficient of friction between the hub and the shaft. For interference fits made by hydraulically expanding the hub with pressurized oil, $0.12 \le \mu \le 0.15$. For shrink or press fit hubs, $0.15 \le \mu$**

The pressure p is used in Eqs. (4.47) to find stresses in each part. For the shaft:

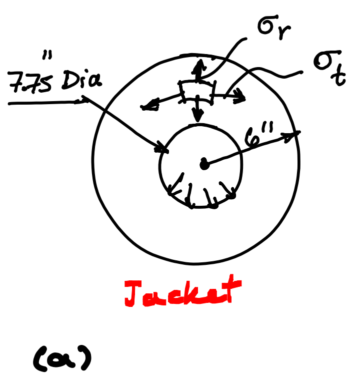
$$\sigma_{t_{shaft}} = -p\,\frac{r^2+r_i^2}{r^2-r_i^2}\ , \quad \sigma_{r_{shaft}} = -p \qquad (10.15a,b)$$

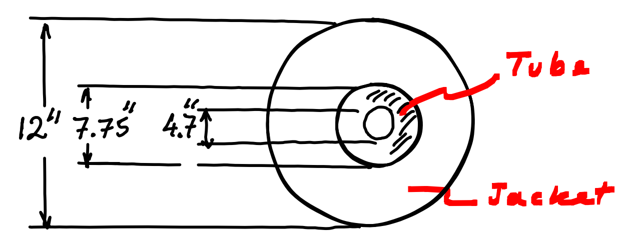
For the hub:

$$\sigma_{t_{hub}} = p\,\frac{r_o^2+r^2}{r_o^2-r^2}\ , \quad \sigma_{r_{hub}} = -p \qquad (10.16a,b)$$

These stresses should be kept below yield, otherwise the hub will become loose on the shaft.

Example: A gun barrel is made by shrinking a steel jacket over a steel tube (E = 29.6 x10$^6$ psi, $\nu$ = 0.3) with dimensions as shown. Determine (a) the diametral interference that will keep the jacket stress at 18000 psi, and (b) the maximum stress in the jacket.




The interference pressure p is related to diametral interference, $\delta$, thru Eq. (10.14a):

$$p = \frac{0.5\,\delta}{\frac{r}{E_o}\left(\frac{r_o^2+r^2}{r_o^2-r^2}+\nu_o\right)+\frac{r}{E_i}\left(\frac{r^2+r_i^2}{r^2-r_i^2}-\nu_i\right)} \qquad (10.14a)$$

In this problem both jacket and tube are made of steel. Hence, $E_o = E_i = E$ and $\nu_o = \nu_i = \nu$, and

$$p = \frac{0.5\,\delta}{\frac{r}{E}\,\frac{2r^2(r_o^2-r_i^2)}{(r_o^2-r^2)(r^2-r_i^2)}}$$

and

$$\delta = \frac{pr}{E}\,\frac{4r^2(r_o^2 - r_i^2)}{(r_o^2 - r^2)(r^2 - r_i^2)}$$

**Note: The amount of delta is aproximately 0.001 units per shaft diameter for larger shafts (e.g., 8") and 0.002 units per shaft diameter for smaller shafts (e.g., 2").**

(6) Max. stress in jacket is $\sigma_{ti} = 18{,}000$ psi

**Stress Concentration due to Interference Fit**

**Even though there is no abrupt change in cross section, an interference fit creates stress concentration in the shaft and hub at the ends of the hub due the abrupt change from uncompressed to compressed material. Determine $K_t$ from Fig. 10-20 and use it in static analysis to check for yielding. For dynamic loading, use $K_t$ to find $k_f$ to use in Eq. (10.8).**

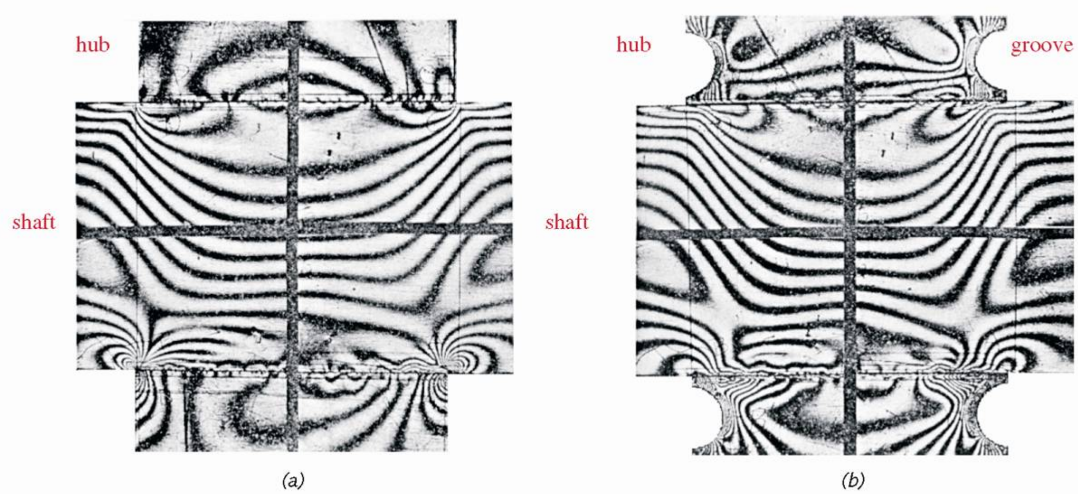
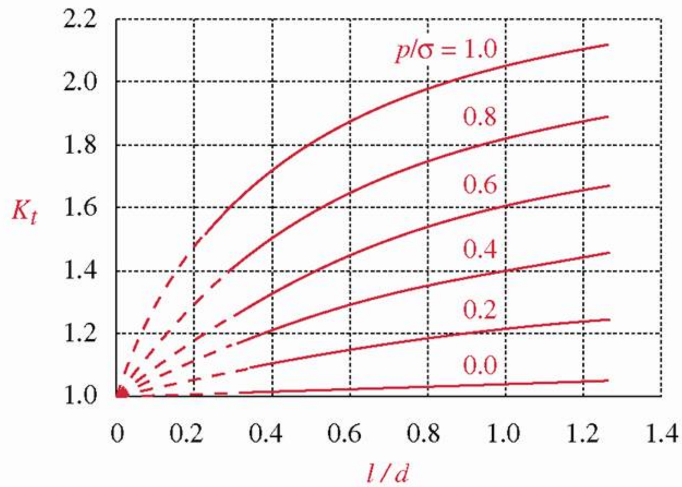



Figure ~~9-19~~ 10-19

Photoelastic Stress Analysis of (*a*) A Plain Press-fit Assembly and (*b*) A Grooved-Hub Press-fit Assembly. Source: R. E. Peterson and A. M. Wahl, "Fatigue of Shafts at Fitted Members, with a Related Photoelastic Analysis," *ASME J. App. Mech.*, vol. 57, p. A1, 1935.

$$p/\sigma = \frac{\text{nominal press-fit pressure}}{\text{nominal bending stress}}$$

$$l\,/\,d = \frac{\text{length of hub}}{\text{diameter of shaft}}$$




10-20

Figure ~~9-20~~

Stress Concentration in a Press-Fit or Shrink-Fit Hub on a Shaft.
Source: R. E. Peterson and A. M. Wahl, "Fatigue of Shafts at Fitted Members, with a Related Photoelastic Analysis," *ASME J. App. Mech.,* vol. 57, p. A73, 1935.

**Reading Assignment: Sections 10.13 - 10.17.**